General Remarks

midterm open book

unlikely: question like the ones under 2

True / False questions on practice exam.

① $\sum \frac{x^{3n}}{\sqrt{n}}$ interval of convergence

radius of convergence: $\limsup \left|\frac{1}{\sqrt{n}}\right|^{1/3n}$

$= \limsup_{n\to\infty} n^{-1/6n} = 1$

(use Hospital for exponent $-\frac{\ln x}{6x}$)

$e^{-\frac{\ln n}{6n}} \sim \left(e^{-\frac{\ln x}{6x}}\right)$

radius of con. $= 1$

does converge at $x = -1$
by alternate series test.

F

② $f_n \to f$ uniformly on $[0,1]$, f_n cont.

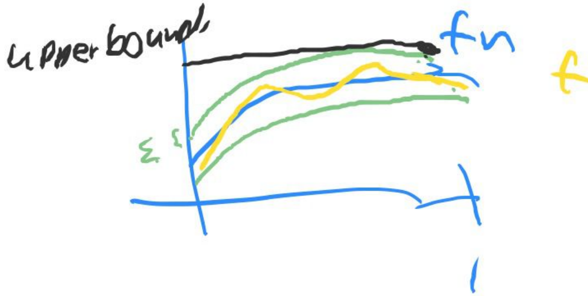
f bounded?

T

(reason: each f_n is bounded (cont function on compact set!)

(f_n) Cauchy sequence

$\Rightarrow$ uniformly bounded:)



for large enough N

$|f_n(x) - f_m(x)| < \varepsilon \ \forall x \in [0,1]$

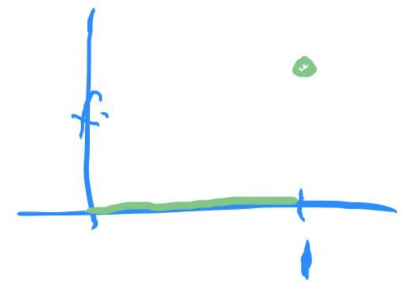
③ $f_n \to f$ pointwise, f_n cont.

is f cont.?

F

$f_n(x) = x^n$

converges to



not cont.

④ $\sum 2^n a_n$ converges

$\overset{?}{\Longrightarrow}$ $\sum a_n x^n$ converges uniformly on $[0,1]$?

reason: know for any power series $\sum a_n x^n$

converges for $|x| < R$

diverges for $|x| > R$

T

$\Rightarrow$ for our series $R \geq 2$ (if $R < 2$ $\sum 2^n a_n$ would not converge

converges uniformly on any $[-R_1, R_1]$, $R_1 < R$

take $R_1 = 1$

5. $\sum a_{2n}$ and $\sum a_{2n-1}$ converge.

Does $\sum a_n x^n$ converge uniformly on $[-1,1]$?

T

reason tricky?

By assumptio $\sum a_n x^n$ converges for

$x=1$ $\quad \sum a_{2n} + \sum a_{2n+1}$

and for $x=-1$ $\quad \sum a_{2n} - \sum a_{2n+1}$

$\Rightarrow$ radius of convergence $R \geq 1$

uniform if $R>1$ ✓

if $R=1$: it follows from Abel's Theorem

⑤ $$\sum_{n=0}^{\infty} \frac{x^{2n}}{2^n n!}$$

ⓐ claim: converges uniformly on $[-M, M]$ $\forall M > 0$

proof. enough to show:
radius of convergence $R = \infty$

$$\Longleftrightarrow \quad \frac{1}{R} = \beta = \limsup_{n\to\infty} \left|\frac{1}{2^n n!}\right|^{1/2n} \overset{!}{=} 0$$

$$\limsup_{n\to\infty} \left|\frac{1}{2^n n!}\right|^{1/2n} =$$

(use trick $y = x^2$

$$\boxed{\sum_{n=0}^{\infty} \frac{y^n}{2^n n!}}$$

✓

can use ratio test. $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim \frac{2^n n!}{2^{n+1}(n+1)!} = \lim \frac{1}{2(n+1)} = 0$

$\Rightarrow$ converges for all y $\Rightarrow$)

$\Rightarrow \quad \sum_{n=0}^{\infty} \frac{x^{2n}}{2^n n!}$ converges for all x.

(b) claim: does NOT converge uniformly?

enough to show:

Consider partial sum $S_k(x) = \sum_{n=0}^{k} \frac{x^{2n}}{2^n n!}$

$(S_k(x))$ not uniform Cauchy.

i.e. for e.g. $\varepsilon = 1$ and for all $N > 0$

exists $k, l > N$ and $x \in \mathbb{R}$

$$|S_k(x) - S_l(x)| > 1$$

Observation: $|S_{k+1}(x) - S_k(x)| = \left| \frac{x^{2(k+1)}}{2^{k+1}(k+1)!} \right| \longrightarrow \infty$ if $x \to \infty$.

$\Rightarrow$ for x large enough

$$|S_{k+1}(x) - S_k(x)| > 1$$

for any $k > N$.

5(c):

$$f'(x) = \sum_{n=1}^{\infty} \frac{2n}{2^n n!} x^{2n-1}$$

$$= x \sum_{n=0}^{\infty} \frac{x^{2n}}{2^n n!}$$

$$= x \sum_{n=1}^{\infty} \frac{1}{2^{n-1}(n-1)!} x^{2n-2 = 2(n-1)} =$$

subst $n-1 \to n$